


# STUDIO DI FUNZIONE COMPLETO

STUDIARE LA FUNZIONE  $f(x) = \sqrt{x+1} - \frac{2}{\sqrt{x-1}}$ .

$$D(f) = \{x \in \mathbb{R} \mid x+1 \geq 0 \wedge x-1 > 0\}$$

LE 2 CONDIZIONI DEVONO VALERE ENTRAMBE:

$$x+1 \geq 0 \Rightarrow x \geq -1$$
A horizontal number line with an arrow pointing to the right, labeled 'R'. A tick mark is at -1, and the region to the right of -1 is shaded with diagonal lines.

$$x-1 > 0 \Rightarrow x > 1$$
A horizontal number line with an arrow pointing to the right, labeled 'R'. A tick mark is at 1, and the region to the right of 1 is shaded with diagonal lines.

INTERSEZIONE



A   $A \cap B = B$   
 $A \cup B = A$

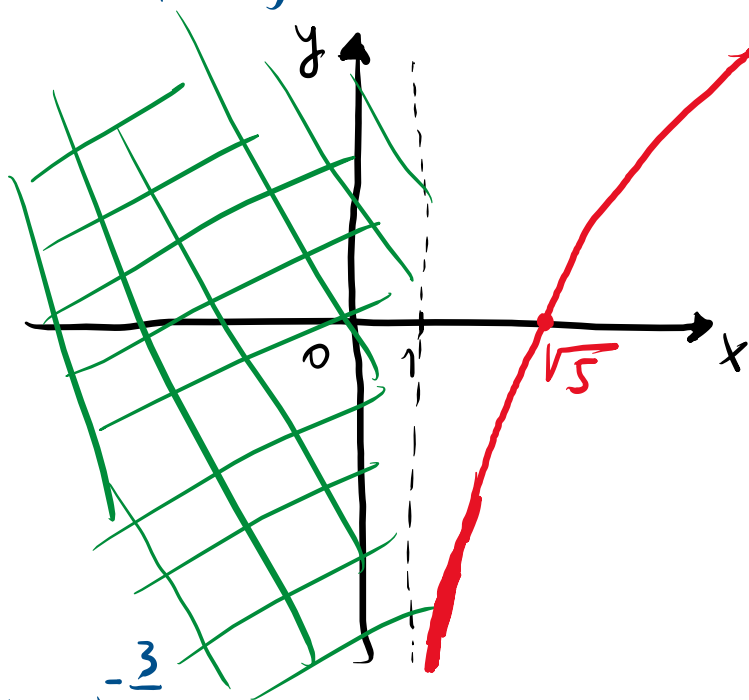
$$D(f) = (1, +\infty)$$

$$\lim_{x \rightarrow +\infty} \left( \sqrt{x+1} - \frac{2}{\sqrt{x-1}} \right) = +\infty$$

$$\lim_{x \rightarrow 1^+} \left( \sqrt{x+1} - \frac{2}{\sqrt{x-1}} \right) = -\infty$$

DERIVATA PRIMA:

$$f'(x) = \frac{1}{2\sqrt{x+1}} - 2 \cdot \left( -\frac{1}{2} \right) \cdot (x-1)^{-\frac{3}{2}} =$$



$$= \frac{1}{2\sqrt{x+1}} + \frac{1}{(x-1)\sqrt{x-1}} = \text{somma di 2 quantità}$$

sempre positive in  $D(f)$ , quindi  $f'(x) > 0 \quad \forall x \in D(f)$ .

$f(x)$  quindi è sempre crescente.

PRO' ESSERE  
CONVEXA      CONCAVA

DERIVATA SECONDA:  $f''(x) = (f'(x))' = \frac{1}{2} \cdot \left(-\frac{1}{2}\right) \cdot (x+1)^{-\frac{3}{2}} +$   
 $+ \left(-\frac{3}{2}\right) \cdot (x-1)^{-\frac{5}{2}} = -\frac{1}{4 \cdot (x+1)^{\frac{3}{2}}} - \frac{3}{2(x+1)^{\frac{5}{2}}} < 0$

IN TUTTO IL DOMINIO.  $f''(x) < 0 \Rightarrow$  FUNZIONE CONCAVA.

IN QUALE PUNTO LA FUNZIONE INTERSECA L'ASSE X?

$$\begin{cases} f(x) = 0 \\ y = 0 \end{cases} \quad \sqrt{x+1} - \frac{2}{\sqrt{x-1}} = 0$$

$$\frac{\sqrt{x+1} \cdot \sqrt{x-1} - 2}{\sqrt{x-1}} = 0 \Rightarrow \sqrt{(x+1)(x-1)} = 2$$

$$x^2 - 1 = 4 \Rightarrow x^2 = 5 \begin{cases} x_1 = \sqrt{5} \Rightarrow f(x) \text{ PASSA PER IL PUNTO } (\sqrt{5}, 0). \\ x_2 = -\sqrt{5} \notin D(f) \end{cases}$$