STUDIO DI FUNZIONE COMPLETO

STUDIARE LA FUNZIONE $f(x) = \sqrt{x+1} - \dfrac{2}{\sqrt{x-1}}$.

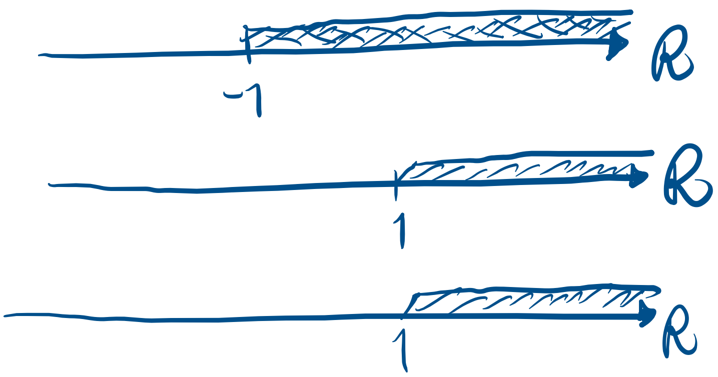
$$D(f) = \{x \in \mathbb{R} \mid x+1 \geq 0 \wedge x-1 > 0\}$$

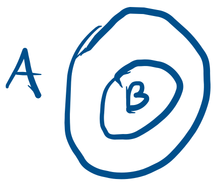
LE 2 CONDIZIONI DEVONO VALERE ENTRAMBE:

$x+1 \geq 0 \Rightarrow x \geq -1$

$x-1 > 0 \Rightarrow x > 1$

INTERSEZIONE





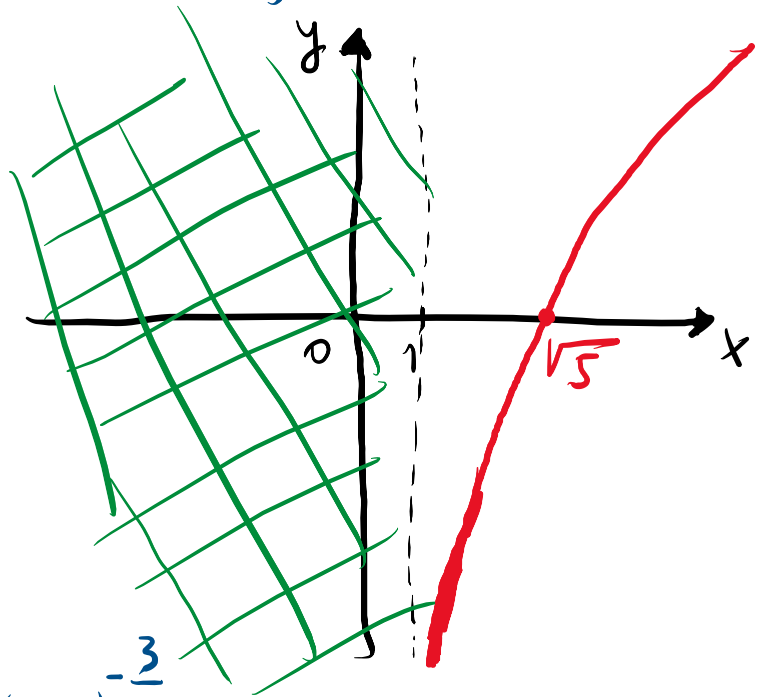
$A \cap B = B$

$A \cup B = A$

$D(f) = (1, +\infty)$

$$\lim_{x \to +\infty} \left(\sqrt{x+1} - \frac{2}{\sqrt{x-1}}\right) = +\infty$$

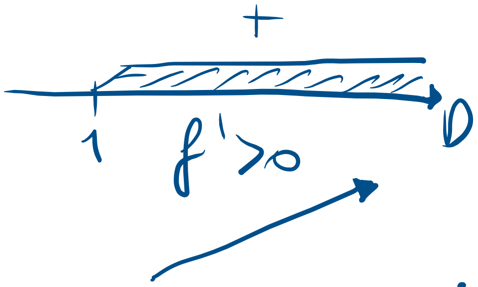
$$\lim_{x \to 1^+} \left(\sqrt{x+1} - \frac{2}{\sqrt{x-1}}\right) = -\infty$$



DERIVATA PRIMA:

$$f'(x) = \frac{1}{2\sqrt{x+1}} - \cancel{2} \cdot \left(-\frac{1}{\cancel{2}}\right) \cdot (x-1)^{-\frac{3}{2}} =$$

$$= \frac{1}{2\sqrt{x+1}} + \frac{1}{(x-1)\cdot\sqrt{x-1}}$$ = SOMMA DI 2 QUANTITÀ SEMPRE POSITIVE IN $D(f)$, QUINDI $f'(x) > 0 \quad \forall x \in D(f)$.



$f(x)$ QUINDI È SEMPRE CRESCENTE.

PUÒ ESSERE

<u>DERIVATA SEGONDA</u>: $f''(x) = (f'(x))' = \frac{1}{2}\cdot\left(-\frac{1}{2}\right)\cdot(x+1)^{-\frac{3}{2}} + \left(-\frac{3}{2}\right)\cdot(x-1)^{-\frac{5}{2}} = -\frac{1}{4\cdot(x+1)^{\frac{3}{2}}} - \frac{3}{2\,(x+1)^{\frac{5}{2}}} < 0$

IN TUTTO IL DOMINIO. $f''(x) < 0 \Rightarrow$ FUNZIONE CONCAVA.

IN QUALE PUNTO LA FUNZIONE INTERSECA L'ASSE X?

$$\begin{cases} f(x) = 0 \\ y = 0 \end{cases} \qquad \sqrt{x+1} - \frac{2}{\sqrt{x-1}} = 0$$

$$\frac{\sqrt{x+1}\cdot\sqrt{x-1} - 2}{\cancel{\sqrt{x-1}}} = 0 \Rightarrow \left(\sqrt{(x+1)\cdot(x-1)}\right)^2 = 2^2$$

$$x^2 - 1 = 4 \Rightarrow x^2 = 5 \begin{cases} x_1 = \sqrt{5} \Rightarrow f(x) \text{ PASSA PER IL PUNTO } (\sqrt{5}, 0). \\ x_2 = -\sqrt{5} \notin D(f) \end{cases}$$