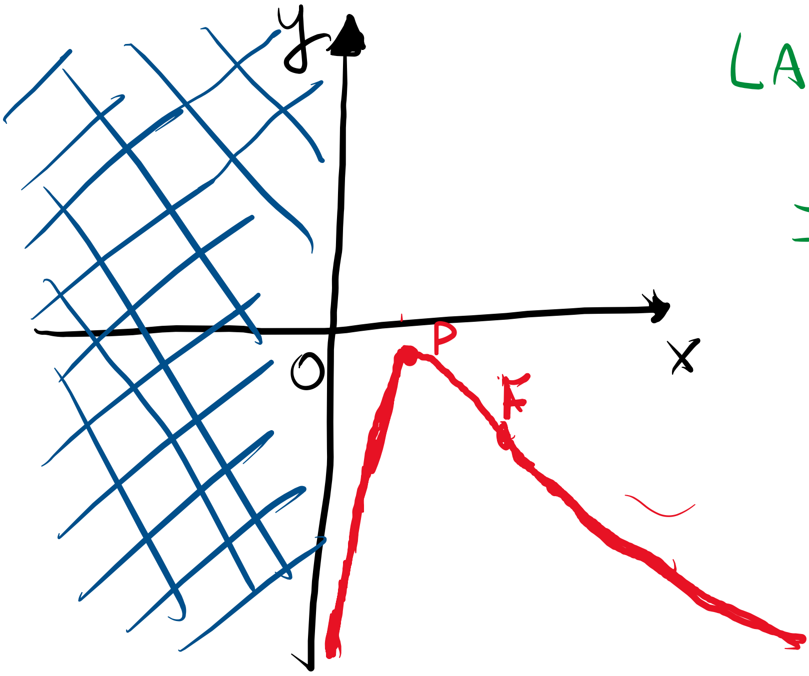


LA FUNZIONE $f(x) = \ln\left(\frac{x}{x^2+1}\right)$ AMMETTE UN ASINTOTO VERTICALE A $x=0$.

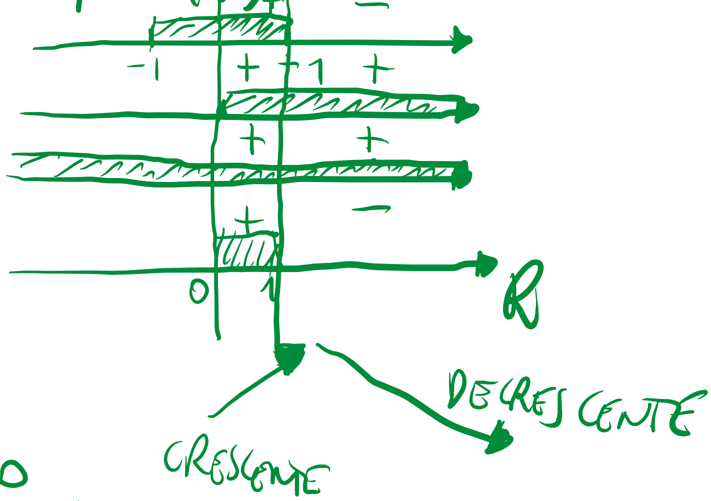
DERIVATA PRIMA: $$f'(x) = \frac{\dfrac{1\cdot(x^2+1)-x\cdot 2x}{(x^2+1)^{\cancel{2}}}}{\dfrac{x}{\cancel{x^2+1}}} =$$

$$= \frac{x^2+1-2x^2}{x(x^2+1)} = \frac{-x^2+1}{x(x^2+1)} \geq 0$$

$f'(x)=0$ SE $-x^2+1=0 \Rightarrow x^2-1=0 \Rightarrow x^2=1 \Rightarrow$

$\Rightarrow x_{1,2} = \mp 1$ ⟨ $x_1 = 1 \in D(f)$; $x_2 = -1 \notin D(f) \Rightarrow x_2$ NON ACCETTABILE

$$f'(x) \geq 0 \quad \begin{cases} -x^2+1 \geq 0 \\ x > 0 \\ x^2+1 > 0 \end{cases}$$



$(1, f(1))$ È UN MASSIMO RELATIVO (ANCHE ASSOLUTO)

$$f(1) = \ln\left(\frac{1}{1^2+1}\right) = \ln\left(\frac{1}{2}\right) < 0$$

$P\left(1, \ln\left(\frac{1}{2}\right)\right) = (1, -\ln 2)$ E' IL MASSIMO.

$$f''(x) = (f'(x))' = \left(\frac{-x^2+1}{x(x^2+1)}\right)' =$$

$$= \frac{(-2x)\cdot(x^3+x) - (-x^2+1)\cdot(3x^2+1)}{(x^3+x)^2} =$$

$$= \frac{-2x^4-2x^2-(-3x^4-x^2+3x^2+1)}{(x^3+x)^2} =$$

$$= \frac{x^4-4x^2-1}{(x^3+x)^2} = 0;$$

$x^4-4x^2-1=0$ E' UN'EQUAZIONE BIQUADRATICA.

SOSTITUZIONE $x^2 = t \Rightarrow t^2-4t-1=0$

$$t_{1,2} = \frac{4 \mp \sqrt{16+4}}{2} = \frac{4 \mp 2\sqrt{5}}{2} = 2 \mp \sqrt{5}.$$

$x^2 = 2-\sqrt{5}$ NON HA SOLUZIONI. $x^2 = 2+\sqrt{5}$

$x_1 = -\sqrt{2+\sqrt{5}} \notin D(f)$

$x_2 = +\sqrt{2+\sqrt{5}}$

$F_2 = \left(\sqrt{2+\sqrt{5}}, f\left(\sqrt{2+\sqrt{5}}\right)\right).$

DOPO IL FLESSO, CAMBIA DA CONCAVA A CONVESSA.